CALCULATION OF TEMPERATURE FIELDS IN CYLINDRICAL BODIES DUE TO MOBILE SOURCES

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1. A Mobile Ring Source

A ring source of width $2h_0$ moves with velocity V over the surface of a solid cylinder of radius R. The source intensity varies with time in a predetermined fashion:

$$q = q(t).$$

The initial temperature of the cylinder is equal to the temperature T_0 of the ambient medium. The thermophysical parameters are assumed to be constant. The problem is reduced to the solution of the equation (in dimensional variables)

$$\frac{\partial\theta}{\partial \mathrm{Fo}} = \frac{1}{\rho}\cdot\frac{\partial\theta}{\partial\rho}\left(\rho\frac{\partial\theta}{\partial\rho}\right) + \frac{\partial^2\theta}{\partial\xi^2} + P\frac{\partial\theta}{\partial\xi}$$

subject to the initial and boundary conditions

$$\theta(\rho,\xi,0) = 0,$$

$$\left.\frac{\partial\theta}{\partial\rho}\right|_{\rho=1} = \begin{cases} 0, & |\xi| > h, \\ K(\mathrm{Fo}), & |\xi| \leqslant h, \end{cases}$$

where

$$\theta = \frac{T - T_0}{T_0},\ \rho = \frac{r}{R},\ \xi = \frac{z}{R},\ h = \frac{h_0}{R},$$

$$\mathrm{Fo} = \frac{a}{R^2}t,\ P = \frac{v}{a}R,\ K(\mathrm{Fo}) = \frac{R}{\lambda T_0}q(\mathrm{Fo}).$$

The successive application of the Fourier and Hankel transforms [1, 2] results in the following solution:

$$\theta(\rho,\xi,\mathrm{Fo}) = \int_0^{\mathrm{Fo}}\left[1 + \sum_{i=1}^{\infty}\frac{J_0(s_i\rho)}{J_0(s_i)}\exp(-s_i^2\tau)\,K(\mathrm{Fo}-\tau)\,E(\xi,\tau)\right]d\tau, \quad (1)$$

where

$$E(\xi,\tau) = \mathrm{erf}\left(\frac{h+\xi-P\tau}{2\sqrt{\tau}}\right) + \mathrm{erf}\left(\frac{h-\xi+P\tau}{2\sqrt{\tau}}\right).$$

The summation is carried out over all the positive roots of the equation

$$J_0'(s) = 0.$$

2. A Mobile End Source

A thermal source with angular size 2β moves with angular velocity ω over the end of a semiinfinite cylinder of radius R. The initial equation in this case is

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$$\frac{\partial\theta}{\partial \mathrm{Fo}} = \frac{1}{\rho}\cdot\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\theta}{\partial\rho}\right) + \frac{1}{\rho^2}\cdot\frac{\partial^2\theta}{\partial\varphi^2} + \frac{\partial^2\theta}{\partial\xi^2} + \Omega\frac{\partial\theta}{\partial\varphi},\quad \Omega = \frac{R^2}{a}\omega$$

and the initial and boundary conditions are

$$\theta(\rho,\ \varphi,\ \xi,\ 0) = 0,$$

$$\left.\frac{\partial\theta}{\partial\rho}\right|_{\rho=1} = 0,\quad \left.\frac{\partial\theta}{\partial\xi}\right|_{\xi=0} = \begin{cases} -K(\mathrm{Fo}), & |\varphi| \leqslant \beta; \\ 0 & |\varphi| > \beta. \end{cases}$$

The solution is found as a result of the same transformations as in the previous section. The result is

$$\theta(\rho,\ \varphi,\ \xi,\ \mathrm{Fo}) = -\frac{\beta}{\pi^{\frac{3}{2}}}\left\{\int_0^{\mathrm{Fo}} K(\mathrm{Fo}-\tau)\exp\left(-\frac{\xi^2}{4\tau}\right)\frac{d\tau}{\sqrt{\tau}}\right.$$

$$\left. + 4\sum_{n=1}^{\infty}\frac{\sin n\beta}{n\beta}\sum_{i=1}^{\infty}\frac{s_{ni}^2 J_n(s_{ni}\rho)E_{ni}}{(s_{ni}^2 - n^2)\,J_n^2(s_{ni})}\int_0^{\mathrm{Fo}} K(\mathrm{Fo}-\tau)\exp\left[-\left(s_{ni}^2\tau + \frac{\xi^2}{4\tau}\right)\right]\cos n(\varphi + \Omega\tau)\frac{d\tau}{\sqrt{\tau}}\right\},$$

where

$$E_{ni} = \int_0^1 J_n(s_{ni}\rho)\,\rho d\rho.$$

The subscript i represents summation over the roots of the equation $J_n(s) = 0$.

NOTATION

r, φ, z are the cylindrical coordinates;
a, λ are the temperature diffusivity and thermal conductivity, respectively;
$J_n(s\rho)$ is the Bessel functions of the first kind.

LITERATURE CITED

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